Deleuze and Mathematics

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Abstract

«Differential calculus…is the algebra of pure thought» (Deleuze 1994:181–2). Here, in an often-overlooked passage of *Difference and Repetition*, Deleuze presents the model of his philosophy of difference inexplicit terms. In this context, calculus is not a simple mathematical tool but the model for the genesis of every actual reality, including thought. This model makes it possible to explain the working of the mind on the basis of sensory experience in order to produce its determinations while also grounding knowledge on a non-phenomenal reality: ideas as differentials of thought.

This article will begin by clarifying how Deleuze draws from modern analysis in order to formulate the paradigm that allows him to ground knowledge on "real conditions of thought" (Deleuze 1988: 23), that is, pre-individual genetic conditions rather than *a priori*, subjective conditions.

Each engendered domain, in which dialectical Ideas of this or that order are incarnated, possesses its own calculus.

Gilles Deleuze

1. The Metaphysics of Calculus and the Differential Genesis of Thought

The fundamental project of Deleuze's philosophy of difference is, as Anne Sauvagnargues has shown (2009), the elaboration of a transcendental empiricism capable of explaining the genesis of thought's determinations, that is, the real rather than the *a priori* conditions of knowledge. This project has an aim similar to that of post-Kantian idealism, but is nonetheless opposed to it, especially its in its Hegelian form based on the notion of identity and non-identity, on the opposition between A and not-A. In fact, Deleuze seeks to explain the genesis of phenomenal reality and the subject based on a dialectic founded on the idea of the *differential* rather than contradiction: "just as we oppose difference in itself to negativity, so we oppose *dx* to not-A to that of contradiction" (1994: 170). Difference, as infinitesimal and indeterminate variation, is not the limit between two given identities but the non-phenomenal condition of all identity engendered as a determined object of thought. It is then matter of placing thought in relation with its spatio-temporal conditions of realization by accounting for its genesis outside of itself to understand the real (non-phenomenal) conditions of the process of individuation of thought. In this sense, calculus provides the model for the processes by which thought determines itself on the basis of its non-phenomenal, on the basis of the differential, the *dx*.

Before entering into the specifics of Deleuze's philosophy of difference, we will first briefly introduce calculus and the philosophical question that it entails in a more general sense, that of the infinitesimal (dx).

Calculus has two basic operations, each the inverse of the other: differentiation and integration. Given a function represented by a curve in a system of coordinates, differentiation involves finding the derived equations that describe the slope of the curve at a point (its tangent), or, the rate of the function's variation at this point. However, Leibniz's and Newton's fundamental discovery (they were working independently) was the technique of integration, which finds the total variation of a function (the area under the curve) based on its instantaneous variation, that is, based on differentials. This technique, which involves the manipulation of infinitesimal quantities, (*dx*, *dy*: instantaneous variations) makes it possible to solve differential equations linking a function to its derivatives. This is the type problem that interests Deleuze: determining a function's curve from its differentials, understood as instantaneous and indeterminate variations, that is, as differences in themselves rather than as differences between two given identities.

When Leibniz and Newton first elaborated the idea of the infinitesimal (an infinitely small variation) it was highly problematic and raised a philosophical question: are these infinitely small things real or merely notional (Deleuze 1994: 176)? Leibniz and Newtown each answered this question differently, with neither of them being able provide a firm basis for calculus: "Newton's mistake, therefore, is that of making the differentials equal to zero, while Leibniz's mistake is to identify them with the individual or with variability" (Deleuze 1994: 172). It should also be noted that for Deleuze, Leibniz's metaphysics arose as a response to questions related to infinitesimal calculus—monads would be based on infinitesimals. Moreover, Leibniz's theory of perception would be based on the idea of integrating tiny unintelligible perceptions. Thus, Leibniz is considered one of the first to have conceived of a philosophy of difference, based on the metaphysical notion of the infinitesimal (we cannot experience infinitesimals, but infinitesimals are the condition of all experience). However, Leibniz was not alone in this enterprise. A host of other philosophers, particularly in the post-Kantian period, based their own "philosophies of difference" on the metaphysics of calculus: Salomon Maimon, Józef Maria Hoene-Wroński, and Jean Bordas-Demoulin. From this "esoteric history of differential philosophy" (Deleuze 1994: 170), which seeks the metaphysical conditions of knowledge in difference understood as *dx* rather than not-A (contradiction), Deleuze sets out to develop his transcendental empiricism using calculus as a model.

Maimon, Wronski and Bordas-Demoulin sought an interpretation of calculus in which infinitesimals had a noumenal, potential or ideal reality, which makes them the non-phenomenal condition of all phenomena. In their systems, difference (*dx*)appears as "that by which the given is given" (Deleuze 1994: 222). In this sense, "difference is not the phenomenon but the noumenon closest to the phenomenon," that is, the reason for the appearance of all phenomenal reality (Deleuze 1994: 222). *dx* does not form any given identity; we cannot experience the infinitely small, and there is no concept that determines the infinitely small. *dx* is not on the order of the phenomenal, although, from a rationalist perspective, any determinate quantity is infinitely divisible. *dx* is therefore on the order of the ideal rather than the phenomenal. The idea, in this sense, is the indeterminate (*dx, dy*) exceeding itself in a movement towards determinability (*dy/dx*) and even a complete determination. Consequently, the idea can be understood as having the character of sufficient reason: the cause of all determined phenomenal reality.

2. The Idea as Problem

The work of Maimon, Wronski and Bordas-Demoulin constitute the origin of the philosophy of difference that Deleuze develops further by incorporating more recent ideas from Bernard Riemann, Henri Poincaré and Hermann Weyl, which were interpreted philosophically Albert Lautman. In his *Essai sur les notions de structure et d'existence en mathématiques*, Lautman discuss Bernhard Riemann's (1826-1866) surfaces, which provide a mathematical example in which an ideal structure creates determined objects.

Riemann surfaces are two-dimensional varieties (multiplicities, manifolds) that enable the uniformization of functions (making them differentiable) that that have ramifications, namely, singularities (poles) where functions have discontinuities. The method for uniformizing a function is called analytic continuation. This procedure determines the circles of convergence for each point on the curve, starting with individual points and proceeding by the differentiation of differentials. Repeating this process of differentiation makes it possible to determine the differential equations that come closer and closer to the curve to be determined. As differentiation produces circles of convergence, we can think of the differential not only as the tangent of the curve but also as a surface, constituted by the accumulation of these circles which make the function determinable. Notably, Riemann surfaces can represent each branch of a function on a "sheet" and connect these sheets to form a continuous surface (fig. 1). According to Weyl's definition, adopted by Lautman, surfaces are formed by connecting neighborhoods where the function is defined (Weyl 1955: 17).

Since the surface can be seen as the differential of the curve, Riemann took an interest in functions that are determinable on a given surface, on which the conditions of a problem should be specified. According to Lautman's explanation, in order to specify the conditions of a problem on the surface—thereby making it possible to be created—two points *a* and *b* should be selected and joined by a line. Once the surface has been divided by this line, giving it its specific topological character, there exists a potential function for which points *a* and *b* are its poles (singularities). It is then possible to uniformize this function to make it determinable over the entire domain. It is therefore this cut between two singularities that transforms the surface into a structure capable of creating, of engendering analytic functions, determinate objects. The surface, as an ideal, indeterminate instance, constitutes a differential problem whose solutions are the functions it engenders. The surface on which the conditions of the problem or singularities are specified is therefore the ideal instance whose "solutions are like the discontinuities compatible with differential equations, engendered on the basis of an ideal continuity in accordance with the conditions of the problem" (Deleuze 1994: 179). Citing Lautman, Deleuze defines the idea as essentially problematic, because "an object outside experience can be represented only in a problematic form" (Deleuze 1994: 169). The ideal differential problem is therefore characterized by "its difference in kind from solutions; its transcendence in relation to the solutions that it engenders on the basis of its own determinant conditions; and its immanence in the solutions which cover it" (Deleuze 1994: 179). These are properties of the virtual: real but not actual conditions of all actual objects (phenomena) that we experience.

3. Ideas are Virtual Multiplicities

As described above, idea-problems are derived topological spaces where the distribution of singularities form the conditions of the problem while also making the determination of solutions possible. The Riemann surface that Lautman uses to explain his theory of problematic ideas is a *manifold*, that is, a multiplicity or variety. Building on this, Deleuze can affirm that "ideas are multiplicities; every idea is a multiplicity or a variety" (1994: 182). We also know that the term multiplicity was used in reference to Riemann by another philosopher who inspired Deleuze: Henri Bergson. As Deleuze writes in *Bergsonism*:

> The word multiplicity is not there as a vague noun corresponding to the well-known philosophical notion of the Multiple in general. In fact *for Bergson is not a question of opposing the Multiple to the One but, on the contrary, of distinguishing between two types of multiplicity*. Now, this problem goes back to a scholar of genius, G.B.R. Riemann, a physicist and mathematician. (Deleuze 1988: 39, italics in original)

We will not delve into the details of *Matter and Memory.* It is enough to emphasize that Bergson's concern was distinguishing between two types of multiplicity: continuous and discontinuous. Discrete multiplicities are "the measure of one of their parts being given by the number of elements they contain," whereas continuous multiplicities "find a metrical principle in something else, even if only in phenomena unfolding in them or in the forces acting in them" (Deleuze 1988: 39).[1] For Bergson, discrete multiplicities, extensive and metric, characterized actual phenomena understood by science, while qualitative multiplicities, continuous and intensive, characterized the temporal, subjective dimension of duration. For Deleuze, the point is not to identify the actual and virtual of two different multiplicities but rather to understand problems in terms of multiplicity so as not to subordinate ideas to the concept of identity, in other words, the One. Ideas are therefore virtual multiplicities, "whether they characterize the multiplicity globally or proceed by the juxtaposition of neighboring regions" (Deleuze 1994: 183). Multiplicities characterized globally are Riemann surfaces, created for analysis and with a specific topological structure depending on how they have been divided by a line as described above. By contrast, multiplicities stemming from a juxtaposition of neighborhoods were created by Riemann as part of his non-Euclidean differential geometry. Following Weyl, Deleuze does not distinguish between multiplicities, which for Bergson marked the two distinctive domains of the actual and the virtual(1955: 17). According to Weyl, both types of varieties or multiplicities can be understood as structures formed by the joining of two local spaces: on one hand the juxtaposition of neighborhoods where the uniformizations of functions are defined and on the other hand the juxtaposition of infinitesimal neighborhoods defined by the value of *ds*. Rather than assigning a different role to the two multiplicities, what matters for Deleuze is to recognize that each multiplicity, as an idea, has a structure consisting of a topological space that is strictly dependent on the internal organization of its genetic elements; what is important is that each multiplicity "is intrinsically defined, without external reference or recourse to a uniform space in which it would be submerged" (Deleuze 1994: 183). A multiplicity, as an idea, is a "system of multiple, non-localizable connections between differential elements which is incarnated in real relations and actual terms" (Deleuze 1994: 183). Multiplicities, as differential conditions of problems, are important for Deleuze because they make it possible to conceive of the Idea without making reference to notions of the one and multiple: varieties are a system of relations between indeterminate elements (*dx*, *dy*) that do not count as one.

4. Singularities

Multiplicities, as ideal problematic structures, contain points that are very important for the determination of solutions: singularities. As Lautman, in a passage cited by Deleuze, stresses, singularities are becoming more and more important in contemporary analysis because:

[1] Note that this definition is taken from Herman Weyl's *Space, Time, Matter*.

> 1.they allow the determination of a of a fundamental system of solutions which can be analytically extended over every path which does not encounter any singularities; 2. …their role is divide up a domain so that the function which ensures the representation can be defined in this domain; 3. they allow the passage from local integration of the differential equations to the global characteristics of analytic functions which are solutions of these equations. (Lautman 1936 v. 2: 133, quoted in Deleuze 1994: 324)

First, singularities are the points that make it possible, by the extension of neighborhoods, to characterize the succession of ordinary points that lie between one singularity and another. Next, singularities are the points that make the surface, or the multiplicity, capable of creativity because they determine the cut that produces analytic functions. Finally, they make it possible to move from the local characterization of functions, that is, by neighborhoods, to their global characterization. This last property of singularities can be understood through the qualitative method introduced into analysis by Henri Poincaré in his *Mémoire sur les courbes définies par une équation différentielle* (1881). This method entails a topological study of curves, that is, geometric representations of integral curves, integrated on the basis of differential equations with complex, undefined values. These values cause series to diverge, making the method of analytic continuation impossible, as it requires the convergence of a point to the neighborhood in order to determine the function. Through the topological study of curves, it is possible to determine the existence of a special type of singularity: essential singularities—"dips, nodes, focal points, centers." These singularities are points around which the values of the function fluctuate without stabilizing. In other words, essential singularities act as attractors that determine the shape of integral curves (Duffy 2013: 40-41).

It should also be noted that these singularities provide an example of the genetic power of the vector field and make it possible to solve the problem posed by complex differentials, whose series do not converge toward an analytic function.

> One thinks in particular of the role of the regular and singular points which enter into the complete determination of a species of curve. No doubt the specification of the singular points (for example, dips, nodes, focal points, centers) is undertaken by means of the form of integral curves, which refers back to the solutions for the differential equation. There is nevertheless a complete determination with regard to the existence and distribution of these points which depends upon a completelydifferent instance – namely, the field of vectors defined by the equation itself(Deleuze 1994: 177).

Singularities thus take on a crucial role in Deleuze's philosophy. Because their distribution defines the potentiality of the topological space of multiplicities they are of fundamental importance for the definition of the problem to be solved.

5. Virtual and Actual

Ideas, as problematic structures, are the sufficient reason for the appearance of phenomena, that is, actual objects in their present being in consciousness. These ideal structures or multiplicities are differen**t**iated (with a "t"). They are characterized by relations between differential elements (dy/dx) and by the distribution of singularities. Like a well-constructed problem, ideas have a structure that makes them determinable, and this determination can only take place during the process of finding a solution: *differen**c**iation* (with a "c"). The differen**t**iated problematic structure, different in kind to *differen**c**iated* solutions, constitutes the domain of the actual. The genesis of determinable ideas, moving from differen**t**iated to differen**c**iated, from problem to solution, therefore moves from the virtual to the actual.

> It is sufficient to understand that the genesis takes place in time not between one actual term, however small, and another actual term, but between the virtual and its actualization – in other words, it goes from the structure to its incarnation, from the conditions of a problem to the cases of solution, from the differential elements to their ideal connections to actual terms and diverse real relations which constitute at each moment the actuality of time. (Deleuze 1994: 183)

The virtual is the idea, or the differen**t**iated multiplicity (manifold). In contrast, the actual is the domain of phenomena produced as solutions: "It is always in relation to a differen**t**iated problem or to the differen**t**iated conditions of a problem that a differen**c**iation of species and parts is carried out, as though it corresponded to the cases of solution of the problem" (Deleuze 1994: 207). However, the virtual and the possible should not be confused because "the virtuality of the Idea has nothing to do with possibility. Multiplicity tolerates no dependence on the identical in the subject or in the object" (Deleuze 1994: 191). The possible is that which already contains the totality of all that can exist, of all that can be experienced, but actualization "does not result from any limitation of a pre-existing possibility" (Deleuze 1994: 211). The virtual is the differen**t**iated multiplicity (neither one nor multiple and therefore non-totalizable, inconceivable as a whole) that is differen**c**iated by allowing solutions that do not resemble the problem's conditions to appear: "actualization or differen**c**iation is always a genuine creation" (Deleuze 1994: 212).

The spatio–temporal dynamism, during which a differen**t**iated idea is differen**c**iated, is a process of individuation in the Simondonian sense of "dramatizing" the idea (Simondon 1964)—an "indistinct differential relation in the Idea to incarnate itself in a distinct quality and a distinguished intensity" (Deleuze 1994: 245). As a result, "the individual finds itself attached to a pre-individual half which is not the impersonal within it so much as the reservoir of its singularities" (Deleuze 1994: 246). This virtual half, a multiplicity of differential relations, remains immanent in the actualized individual and constitutes the pre-subjective transcendental plane which is the condition of successive differen**c**iations. This procedure of differen**c**iation, or actualization, makes it possible for individuals emerge as solutions. But it should be noted that each individuated being is already individuated as consciousness: "Every spatio-temporal dynamism is accompanied by the emergence of an elementary consciousness which itself traces directions, double movements and migrations, and is born on the threshold of the condensed singularities of the body or object whose consciousness it is" (Deleuze 1994: 220).

The subject is therefore constituted in relation to the idea and does not preexist the problem that its consciousness resolves by integration of differential conditions, "for we are never fixed at a moment or in a given state but always fixed by an Idea as though in the glimmer of a look, always fixed in a movement that is underway" (Deleuze 1994, 219). The process of actualization entails a process of individuation that simultaneously concerns beings and their consciousness. Thought is not endowed *a priori* with a transcendental structure that belongs to an always-already constituted subject. Rather, it is determined during the process of differentiation, of actualization in relation to the problematic conditions that are the condition of any spatio-temporal dynamism of individuation.

6. Deleuze and the Mathematics of Mind

The transition from the virtual to the actual is thus based on the model of differential calculus: beings are determined as solutions to the problem posed by ideal multiplicities. This determination also entails the determination of a larval consciousness—an individuation that corresponds to the same differential conditions, while at the same time an individuation that differs in nature from them. Therefore, thought, as differentiated degree from larval consciousness, is determined by a sort of integration of differentials that are felt as intensive differences: experience makes us "feel" the problematic conditions that force thought to think, to determine itself as a solution. Although it is a simple mathematical tool, differential calculus "finds its sense in the revelation of a dialectic which points beyond mathematics" (Deleuze 1994, 179) and is therefore valid as a model of the working of the mind that meets the requirements of a transcendental empiricism. The philosophy of difference, according to which the indeterminate *dx* renders itself determinable as a differential relation within an idea-problem, forms a dialectic that governs all processes of actualization: "In this sense there is a *mathesis universalis* corresponding to the universality of the dialectic" (Deleuze 1994, 181). There are problems in mathematics, physics, biology, sociology, psychology whose solution is found in these different disciplines, actualizing specific solutions in thought. But in each case, each solution is determined in relation to "a system of connections between differential elements, a system of differential relations between genetic elements" (Deleuze 1994, 181). In other words, "each engendered domain, in which dialectical Ideas of this or that order are incarnated, possess its own calculus" (Deleuze 1994, 181). It should be noted that for Deleuze, analysis does not represent the most highly developed mode of mathematical expression and that mathematical problems give rise to solutions in other areas, such as set theory or topology. In any case, calculus forms the dialectical model of the workings of the mind and "has a wider universal sense in which it designates the composite whole that includes Problems or dialectical Ideas, the scientific expression of problems, and the establishment of fields of solution" (Deleuze 1994, 181). In short, "if Ideas are the differentials of thought, there is a differential calculus corresponding to each idea, an alphabet of what it means to think" (Deleuze 1994, 181).

This way of thinking about calculus recalls Lautman's theories, which provided a Platonic interpretation of analysis and mathematics in general. According to Lautman, mathematical objects are engendered through a sort of dialectic that operates by division or decomposition on the basis of ideal structures. However, it should be noted that, in contrast with Lautman, Deleuze regards these idea-problems as immanent to any type of solution, not only mathematical solutions. Additionally, these idea-problems are in the process of becoming. The distribution of singularities that distinguishes ideal multiplicities changes on the model of a dice game: unpredictable throws of the dice are at the root of the variable definition of the conditions of a problem:

> We asked what was the origin of Ideas and where problems come from: in reply we invoke throws of the dice, imperatives and questions of chance instead of an apodictic principle; an aleatory point at which everything becomes *ungrounded* instead of a solid ground. We contrast this chance with arbitrariness to the extent that it is affirmed, imperatively affirmed in the particular manner of the question; but we measure this affirmation itself by the resonance establish between the problematic elements which result from a throw of the dice. (Deleuze 1994, 200)

To build his transcendental empiricism, legitimizing scientific knowledge as true on the basis of the relative's access to absolute truth, Deleuze uses Lautman's philosophy of mathematics as his model. In this way, any theory of mathematics has a value that is relative to specific problematic circumstances: the history of mathematics is a history of problems.

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