A Fistful of Differencials

by ALESSANDRO SARTI & GIOVANNA CITTI

Abstract

What is the dynamics underlying the constitution of a new form? Reconsidering the original Deleuzian-Guattarian idea of singular becoming and using contemporary instruments of geometric analysis, we introduce the concept of "differential heterogenesis" as a mathematical framework to envisage the emergence of singular forms from the assemblages of heterogeneous operators. In opposition to the kind of differential calculus that is usually adopted in mathematical-physics, which tends to assume a homogeneous differential equation applied to an entire homogeneous region, heterogenesis allows differential constraints of qualitatively different kinds in different points of space and time. We will discuss the impact of this approach to the individuation of living, perceptual, semiotic and ecological forms.

Introduction

The question we are interested in concerns forms, or rather the becoming of forms. We care about it as mathematicians but in the spirit of 20th century French philosophy, from Gilbert Simondon to Gilles Deleuze. In this context, the becoming of forms has a generative character, deployed as a passage from an intensive plan to its expression in actual forms extended in space and time. For Gilbert Simondon it is a passage from a pre-individual intensive plane to the plane of individuated forms (Simondon 1995). For Gilles Deleuze in *Difference and Repetition*, becoming is the passage from a virtual plane to an actualized one (Deleuze 1994), where forms are deployed in extended space and time. Deleuze differs from Simondon in explicitly equipping this passage with a differential calculus where the evolution of forms is the solution to a differential problem. The idea of Leibniz's differential calculus is revitalized so that the becoming of forms is now viewed as the solution of a distribution of differential constraints that populate the virtual. The virtual, then, is a multiplicity of differential generators that are the genetic elements of every morphodynamics.

The entire process of becoming has to be conceived as a transformation from one multiplicity to another: "Thus each individual is an infinite multiplicity, and the whole of Nature is a multiplicity of perfectly individuated multiplicities" (Deleuze & Guattari 1987).

Already in *Difference and Repetition*, the primary model for this differential multiplicity is the Riemaniann manifold:

> Ideas are multiplicities: every idea is a multiplicity or a manifold. In this Riemannian usage of the word 'multiplicity' (taken up by Husserl, and again by Bergson) the utmost importance must be attached to the substantive form: multiplicity must not designate a combination of the many and the one, but rather an organisation belonging to the many as such, which has no need whatsoever of unity in order to form a system. (Deleuze 1994)

This analogy with manifolds is reinforced in *A Thousand Plateaus* (Deleuze & Guattari 1987), where the heterogeneous character of Riemannian covering is outlined following the interpretation of the mathematician Albert Lautman:

> In short, if we follow Lautman's fine description, Riemannian space is pure patchwork. It has connections, or tactile relations. It has rhythmic values not found elsewhere, even though they can be translated into a metric space. Heterogeneous, in continuous variation, it is a smooth space, insofar as smooth space is amorphous and not homogeneous. (Deleuze & Guattari 1987)

In this context multiplicity is already a heterogeneous composition of elements in continuous transformation: "each multiplicity is already composed of heterogeneous terms in symbiosis, and that a multiplicity is continually transforming itself into a string of other multiplicities" (Deleuze & Guattari 1987).

The heterogeneous character of these differential assemblages is outlined primarily in opposition to structures. Structural positionality is questioned via a strong critique of the concept of structure as a set of potentials that drive various dynamics towards stabilisation.

More or less explicitly, then, the critique is oriented towards structural morphodynamics as homogeneous and stabilized, to which Deleuze and Guattari oppose the dynamics of heterogeneous fluxes in continuous recombination. The concept of the rhizome itself refers to an intensive heterogeneous assemblage allowing the emergence of non-stabilized flows. This represents the main passage towards post-structural dynamics that Deleuze and Guattari calls heterogenesis. In heterogenesis the becoming of forms no longer emerges from generators that are homogeneous in space and time, as in mathematical physics. Instead, the possibility of the mutation of laws is introduced. Heterogenesis, then, is the dynamism that allows for the generation of new forms from continuously changing constellations of differential constraints.

It is interesting to note that, parallel to the Deleuzian elaboration, an independent yet analogous process of heterogenization of spaces emerged from the field of mathematics, beginning with the works of Riemann. The necessity of building sophisticated differential structures led to the introduction of a new direction of research called *geometric analysis*, the main focus of which is to define mathematical tools that can be used to widen the perspective of differential calculus. In particular, there is a relation between the tools produced by modern geometric analysis, which define differential calculus in weaker and weaker settings, and

the idea of looking at dynamics as emergent properties of heterogeneous constraints that differ in space and time.

Here, we deal precisely with this question: how do we rethink the differential in order to produce a heterogenetic becoming of forms. We began to pose these questions in a recent article *Differential Heterogenesis and the Emergence of Semiotic Function* (Sarti, Citti & Piotrowski 2019), where we traced some possible directions of research without presenting exhaustive answers.

A step back: Structural morphodynamics

Of course, the question of heterogenesis is part of the history of the concept of morphogenesis, which starts from the theoretical framework developed by Goethe, who introduced the word itself *morphogenese*, and continues through, for example, the elaborations of D'Arcy Thompson in his fundamental work *On growth and form* (Thompson 1917) and Alan Turing in his seminal paper *The chemical basis of morphogenesis* (Turing 1952), up to the formalization provided by René Thom's catastrophe theory in *Structural Stability and Morphogenesis* (Thom 1985).

The particular operation we would like to tackle here shares some common aspects with the one accomplished by René Thom and Jean Petitot in the 70s-80s, even if this was applied to different materials and dynamics. In those years, the structural paradigm sustained by Lévi-Strauss in cultural anthropology, Jakobson in linguistics, Saussure and Greimas in semiotics and Tesniere in phonology, among others, had already been brought to fulfilment, at least in its philosophical deployment. In respect of this, the intervention of Thom-Petitot (Petitot 1985 [2004]) has to be considered a true translation of structuralism into dynamical terms. Dynamic structuralism in fact interprets the theory of structures by means of the catastrophe theory of René Thom in such a way that structures become dynamical devices that can be controlled in their possibility spaces.

In this way the semiotic square of Greimas becomes a catastrophe with four possible stable dynamics (Petitot 1977), while Lévi-Strauss's canonical formula of the structure of myth becomes a catastrophe with eight stable dynamics (Petitot 1988). More recently, David Piotrowski has proposed to model the Saussurean sign as a catastrophe with two stable dynamics (Piotrowski 2017). By means of the control of dynamics it is possible to move from one side of the semiotic square to the other, or to pass from one state to another in the dynamics of the myth, or from one sign to another in semiogenesis. It is similar to a puppet show, with suitable parameters controlling dynamics just as wires control puppets. Notice, however, that if structural dynamics allows for an essential heterogeneity in the actualized solutions, it is restricted to homogeneous virtuals, meaning that forms are generated by homogeneous differential constraints. A typical example of this is the symmetry-breaking of

solutions in phase transitions, where a different solution is selected via the change of some parameter without changing the differential equation, that is without touching the virtual.

Heterogenesis as an immanent dynamics

After Foucault's harsh criticism of structuralism as a relational system of empty and interchangeable places, the idea of the becoming of forms was drastically transformed in the 80s thanks to the exceptional work of Deleuze and Guattari (1987). Here the concept of heterogenesis becomes a device able to instantiate infinite dynamics thanks to a virtual in continuous recombination. But, as we mentioned, this philosophical paradigm has not yet been translated into epistemic terms, i.e. rewritten in the terms of material dynamics. Today we can try to provide epistemic depth – by means of mathematical research and experimental modelling – to this conceptual elaboration that we could provisionally refer to as a form of post-structuralism.

It is important to reject the accusations of superficiality and irrationality that have been levelled against this paradigm, and to show that heterogenesis puts in place material dynamics that are fundamental to life and the human sciences, and which could be the foundation of an incipient political ecology.

When we discuss post-structural dynamics we are dealing with the becoming of forms that are neither structures nor chaos; between chaotic dynamics on the one side and structures on the other, there is something else. Specifically, between symbolic, biological and semiotic structures, and the complete absence of form, is something interesting: a rich becoming of forms that exceeds structures, changes laws and recombines existing dynamics. We would like to think about the emergence of these forms and the conditions of their emergence, and to explore the mutations of the virtual that are responsible for dynamic deployments. Distinct from the morphodynamics issuing from mathematical physics or structuralism, said dynamics are characterized by a heterogeneous virtual defined by a multiplicity of differential constraints that vary in space and time. In fact, the continuous evolution of differential constraints that is typical of, for example neural plasticity, seems to prevent the introduction of eternal laws such as conservation laws in mathematical physics. In this case, the evolving dynamics could even induce change in the nature and properties of the space in which the dynamic is defined. This is very different to physical laws that are defined by an equation or system of equations that holds homogeneously, always and everywhere, within defined domain! If heterogeneity belongs to the virtual and not just to its actualisation in extended forms (which was already present in structural dynamics), this explains its intensive nature. Heterogenesis operates within the virtual, on the generative plane that Simondon calls pre-individual, because it precedes any individuation of forms. It is this assemblage of heterogeneous differentials and their continuous recombination that generates new and

specific dynamics that we will call singular dynamics, given that their character is not attributable to known forms.

The weakening of differential constraints in geometric analysis

The question of heterogenesis also forms part of the history of mathematics, particularly the history of geometric analysis. If the enormous contributions of Riemann opened geometry to non-Euclidean settings, introducing fundamental anisotropy to space, it became clear, starting with the works of Bony and Hörmander in the 1960s, that it was possible to further remove some of the constraints imposed by the Riemannian differential calculus. In the new geometry, known as sub-Riemannian, the allowed directions of dynamic propagation are described by vector fields that differ from point to point and give rise to continuously changing planes of propagation called admissible planes.

In order to ensure that each point of a space can be connected to any other, Hörmander (1967) imposed a celebrated condition on these vector fields expressed in terms of their commutators, which are new vector fields generated by those given. Of particular importance here are the geometric properties of distance in these spaces as studied by Nagel Stein and Waigner (1985), since the propagation of every dynamic is shaped by the distance function induced by the metric. Here, Rothshild and Stein (1976) faced the specific problem of defining a geometry starting from vector fields that differ from point to point. Since the differential constraints defined by the admissible planes differ from one point to another, they introduced a higher dimensional space called a lifting, whereby the differential constraints at different points can be related, and whose projection at every point is the admissible plane at that point. As a result, dynamic propagation takes place in the lifted space and can be reprojected as a dynamic that differs from point to point, allowing for the interaction of different geometries. Starting with these celebrated results, a new research direction has emerged that allows for increasingly heterogeneous geometric constraints. The most significant results obtained for linear and non-linear operators have been collected in L. Hörmander (2009), while vector fields depending on the solution itself have been considered by L. Capogna, G. Citti and M. Manfredini (2010).

These tools have been recently applied to the modelling of the connectivity of the visual cortex that is at the base of the constitution of perceptual forms, starting from the visual stimulus. Specifically, cells in visual areas are characterized by their ability to select differential features of an image and to propagate this information in a very specific way within a sub-Riemannian geometry. Within a single population of cells, behaviour is homogeneous, and cells are able to select and process a single visual feature in a sub-Riemannian geometry generated by fixed vector fields. In this applications general sub-Riemannian theory has been reduced to the symmetrized setting of Lie groups, as in the proposals by Hoffmann (1989),

Petitot and Tondut (1999), Citti and Sarti (2006) and Sarti et al. (2008). In these early examples of neurogeometry the underlying theory had not yet been fully applied – geometric heterogeneity was only introduced in M.Favali et al. (2017), which takes into account a variety of differential features like boundary orientation, T-junctions and L-junctions.

At this point let's note that this approach of weakening constraints concerns primarily the geometry of propagation, while the homogeneity of the dynamic constraint remains intact. This means, for example, that Rothschild and Stein consider only diffusive dynamics at each point, even if different propagation directions are allowed. To overcome these limitations, a new area of research around radical dynamical/geometrical heterogeneity has opened up.

Differential heterogenesis

Let's consider some differential constraints that pose differential problems in terms of constellations of heterogeneous operators. The heterogeneity of the operators will here be understood from two different perspectives concerning local dynamics and local geometry respectively. Through these perspectives we freely interpret the concept of heterogeneous assemblages (*agencement*) introduced in *Thousand Plateaus* and mathematical concepts from contemporary geometric analysis in order to provide a new description of differential problems in heterogeneous settings.

We find a first level of heterogeneity in the constitutive difference of differential constraints, which can induce a variety of dynamic behaviour that changes from point to point.

The first level of heterogeneity can be formally expressed as the choice of an operator of a different order at each point. The simplest example here is an operator with advective behaviour at one point and diffusive behaviour at others. Advective component are first order terms, have a fast propagation rate and tend to keep the shape of the initial datum. Diffusive operators, which are expressed as second order operators, have a slower propagation rate and a strong smoothing effect. More generally, a constellation of operators with totally different dynamics at every point will be considered.

A second level of heterogeneity is also present since each differential constraint has its own structure of tangent planes constituting the phase space. This characterization of the space is obtained by reinterpreting the admissible tangent planes of Rothschild and Stein and the "plateaus" introduced in *Thousand Plateaus* as the planes that fluxes are allowed to flow on.

Given this continuously changing geometry of flow directions and local differential operators, the heterogeneous differential problem is defined in terms of a composition of the differential constraints on assemblage formation. How this heterogeneous composition is possible is the mathematical problem we have examined in A. Sarti, G. Citti and D. Piotrowski (2019). Heterogeneous assemblages are not here built on the basis of logic compatibility or

compliance, but by the possibility of creating new spaces and dynamics not given *a priori*, in such a way that phase spaces as well as dynamics are invented by the intrinsic construction of the singular composition.

For example, if two operators are defined on open sets with non-empty intersection, two geometries and two dynamics are defined in the intersection. A new space is defined in the intersection via a lifting process, driven by the differential structure of the vector fields generating the local 'plateaus' at every point and their commutators. Commutators interpret in a formal way the differences of differences that are so important in the Deleuzian construction of assemblages. The commutators do not exist separately in each of the lifted operators, but are defined by the interaction which, as a consequence, becomes much more than the simple union of the two collections of vector fields. The size of these spaces necessarily transcends the dimensionality of the initially given spaces and the interaction between them changes the structure and geometry of the interacting objects. *A new space of possibility (phase space) is then created by the assemblage.*

The spatially and temporally varying definition of differential constraints is quite far from the usual differential calculus of mathematical physics, in which the distributions of operators are spatially and temporally homogeneous. Hence mathematical physics can be obtained as a special case when the same operator holds in every spatio-temporal point. The *a priori* character of mathematical physics, where the space is assigned, is completely reversed in the composition of heterogenetic assemblages, in which operators are primary and define the dimensions and qualities of the space: every new operator composed within the constellation completely redefines the entire space of the assemblage. Heterogenetic composition is poles apart from universal laws and lays the conditions for an immanent morphogenesis that is created moment by moment by the assembling of singular concatenations.

In mathematical physics, operatorial homogeneity and the fixity of differential constraints determine the universality of laws and the nomological character of differential models. Notice that if the assemblage of operators is in turn considered as a new differential operator, heterogenesis can be viewed as a morphogenesis of the assemblage operator. A heterogenetic becoming can then be considered as a concurrent morphogenesis of operators, of spaces and of forms in spaces, a concept that is unprecedented in physical and structural dynamics.

To allow for the construction of assemblages, two temporal scales or axes are present. The first is the axis of the actualisation of differential constraints. It is the axis of Chronos common to mathematical physics. The second is the axis Deleuze calls Aion, on which takes place the recombination of differential constraints in new assemblages. On this axis we have a true plasticity of the virtual, meaning the possibility to recombine genetic elements in order to create singular dynamics. Any specific recombination has to be thought of as an exploratory action, closer to a Dada performance than to a finalised process.

The composition of a singular assemblage is then an invention – the creation of new dynamics instant by instant. The inventive character of the assemblage is due to the fact that the space it creates is much more than the union of identitary spaces of single operators. As we clarified in our mathematical presentation, this is because second order differences (differences of differences) increase the dimensions of the tangent space and open to new planes that were previously inconceivable.

This feature reflects the Deleuzian position that rather than searching for the common in the different (in a process of homogeneisation of existing spaces), it is more useful to think in a differential way about difference. Precisely because of these differences of differences (that occur through the mathematical operation of commutation), new spaces arise with all their possible dynamics.

In this context repetition has a very different value to the one it has in Galilean physics. Physical truth lies in the repetition of a physical experiment giving the same result, but any inventive dynamic, like the introduction of an idea for the first time, makes a creative act no longer reproducible. Once the concept is known, its repetition becomes mere use without fascination.

The problematic dimension of becoming

What is the utility of reconsidering heterogenesis from the morphodynamical point of view? The first motivation relies on the fact that the very origin of Deleuzian heterogenesis has an operational nature, since Deleuze takes the Leibnizian differential calculus as a model and, more generally, the operational disposition of Baroque culture. Differential calculus is at the basis of the idea of becoming in *Difference and Repetition* and becoming assumes from the beginning a problematic dimension, in the strict mathematical sense of posing a problem. Deleuze explicitly explains the role of mathematics in its constructivist empiricism:

> […] how can something be given to a subject, and how can the subject give something to itself? Here, the critical requirement is that of a constructivist logic which finds its model in mathematics. The critique is empirical when, having situated ourselves in a purely immanent point of view, which makes possible a description whose rule is found in determinable hypotheses and whose model is found in physics, we ask: how is the subject constituted in the given? The construction of the given makes room for the constitution of the subject. The given is no longer given to a subject; rather, the subject constitutes itself in the given. (Deleuze 2001)

Becoming is viewed as the creative principle arising from a problem defined in terms of a constellation of heterogeneous differential operators. This phase of plastic composition of differentials situates the problematic and intensive dimension of becoming, which can be

regarded as a form of plasticity of the virtual. Mathematics can then be used as a language to evoke the dynamical becoming of a complex materiality endowed by its substantial consistency as a vital, singular, semiogenetic flow. This is similar to the vital materialism of Rosi Braidotti (2002), Isabelle Stengers (2011) and Donna Haraway (2016), but seen from the perspective of the virtual conditions of its deployment.

Beyond this intrinsic motivation, there is also a historically contingent factor that motivates us to elaborate heterogenesis mathematically. In Albert Lautman's epistemic view, mathematics is considered a language that is always relative to specific and situated problematic circumstances in which an important part of mathematical invention consists of the formulation of problems. The history of mathematics is thus considered to be a history of problems more than a history of automatic progress independent from cultural and historical contexts, as is the case in the axiomatic perspective. The work of mathematicians is therefore to envision the entire problematic dimension in an original way.

We are thus interested in the question of heterogenesis primarily in order to problematize rather than to offer solutions. In particular, we are interested in problematizing contemporary models in the life sciences and human sciences. Models in life and human sciences, from the cognitive to the social, from the aesthetic to the semiotic, emerge from a culture of physical science that assumes an invariant and homogeneous distribution of operators. This nomological use of operators is at the base of contemporary modelling culture: the Navier-Stokes equation for viscous fluids, for instance, is the same in all points of space and time. Analogously, Alan Turing's (1952–1992) equation of morphogenesis, also deeply studied by Réné Thom, presents spatial and temporal symmetries.

Within the domain of the life sciences, a serious criticism of invariances and symmetries has been made by Giuseppe Longo (Bailly & Longo 2011; Longo & Montevil 2014), who has outlined the necessity of evolving phase spaces. A similar criticism should be addressed to models of mathematical and computational economics that are based on the interaction of individuals endowed with the same space of rationality. These approaches are founded more or less explicitly on the paradigm of methodological individualism (Laurent 1994; Petitot 2015) in which every process of individuation is reduced to a functional interaction between already individuated units.

While homogeneous constraints can usefully describe swarm intelligence or crowd behaviour, they reduce dynamics to automatisms by excluding any imaginative or creative aspects. We aim to problematize the procedure of homogenization dominant in the life and social sciences and to outline the dynamical heterogeneity of life and its affective, semiotic, social and historical aspects. The purpose of this task is to free up dynamic becoming from any form of unitary and totalizing symmetry and to develop forms, action and thought by means of dispositives of proliferation, juxtaposition and disjunction.

Life, brain, micro-history

Heterogenetic dynamics pertain to different empirical basins. If the empirical basin of the dynamic structuralism of René Thom and Jean Petitot is embryogenesis, that is, the set of dynamics at the core of the formation of biological bodies whose symmetry breaking is controlled by a parameter space, the empirical basin of post-structural dynamics is the brain. The brain is the ultimate Body without Organs, the body that, thanks to plasticity, changes its rules dynamically and rebuilds itself continuously in a situated way. There is thus a necessity to model cerebral dynamics in the most heterogeneous way possible. The brain is made up of neural populations with heterogeneous dynamics that are mathematically described by heterogeneous operators. At the same time, neural populations act on sets of neurochemicals such as neurotransmitters, messengers and neuromodulators that give rise to a heterogeneity of formed substances. Here again, the neural connectivity that defines the structure of the tangent planes of various dynamics differs from population to population. These populations are concatenated in the form of assemblies, at which point they must be considered as material implementations of heterogenesis. Finally, neural connectivity is plastically modified by learning processes that implement a true plasticity of the virtual, which corresponds to a continuous reorganization of the differential rules underlying dynamics. Brain heterogenesis therefore constitutes the material support of every phenomenology of perception and imagination whose forms are deployed as the solution of suitable differential problems (Deleuze and Guattari analyse this topic in their last work *What is philosophy?* [1994]).

On the other hand, we can find post-structural dynamics in the life science when we consider the evolution of living forms on the axis of phylogenesis, along which genetic elements are recombined. Additionally, in this case we have a double temporal axis: the axis of ontogenesis on which living forms are actualized, and the axis of phylogenesis on which generative constraints are recombined. Here we are back at the two temporal axes of post-structural dynamics: the axis of Chronos that allows for actualisation and the axis Deleuze terms Aion, on which the recombination of differential constraints forms new assemblages, new configurations.

Post-structural dynamics are also present in historical becomings – the perspective of micro-history, for example, which teaches us to look at histories in terms of the dynamics of forms and the becomings of morphologies, as traced by Goethe and Walter Benjamin. Micro-historical dynamics (Ginzburg 1980; Gribaudi 2014) are a laboratory for a morphology of multiplicities against the forms of contemporary historiography that present history as a progressive development of global phenomena, uniformly characterizing the whole of a society from their supporting structures to their symbolic and relational forms. This reduction is implemented both in space and in time: in space the same phenomena would be uniformly present in the whole of the society and in time the same logics would unfold in epoch-long

timeframes. Indeed, we should reconsider and reactualise the heterogeneity of forces and the variety of syncretic assemblages that are at the origin of historical dynamics. Rather than a provisional quantitative model, heterogenesis is a morphological device for gaining a qualitative understanding of the generation of forms. We argue that heterogenesis helps us to understand the morphologies of imaginative, historical and phylogenetic becoming. For this reason it is also at the centre of the question of the emergence of meaning in relation to the automatism of information processing.

The flow of expression and its polarisation

What is the difference between information processing and the production of meaning? Let's use an example. For years, we have studied and modelled cerebral processes in terms of information processing even if, it is true, we have done so in a much more nuanced way than the cybernetic cognitivism of the '60s. Today, contemporary techniques of convolutional deep learning allow us to construct AI architectures starting from databases of stimuli. Even in this case, however, it is really just a sophisticated form of information processing. In fact the network is doing nothing more than reformatting the statistics on which it is completely dependent. What is experimentally observed in cerebral dynamics is radically different from this. Cerebral morphologies depend not only on external stimuli but also on the presence of the situated body, both the cinematic-dynamic body with its mechanical constraints as well as the warm body with its large regulation systems linked to sexuality, nutritional circuits, feelings and emotions, the main research area of Spinozist neuroscientist Antonio Damasio. The presence of the body modulates cerebral morphologies by means of mechanisms of reinforced learning in such a way that only the morphologies reinforced by embodied feedback remain active. Cerebral circuits are then selected on the basis of their meaningfulness for the situated body. The result is a highly heterogeneous neural architecture wherein a huge variety of neurochemical dynamics are assembled with different geometries of connectivity.

Here we can touch on the Thomian theory of meaning, on which the French mathematician founds his own physics of sense in a much less structural way than catastrophe theory.

The theory that significant forms are constituted when the body pregnancies take and modulate the salient forms.

The power of reinforced learning goes far beyond the automatic response to Pavlovian stimuli. It is demonstrated, for example, by Patrizia Violi, a semiotician of the Umberto Eco school, in a series of very interesting works about the emergence of primary semiosis in the relationship between the mother and the newborn child. Here semiosis is far from constituting a form of behaviourism since its deployment already shows already a deep cultural and

singular character and semioses are pre-symbolic but already trans-individual and social, irreducible to any disembodied information processing.

We can now come back to the brain and neuromathematics. Brain systems constitute the virtual, the differential, the intensive and the generative plan of any cognitive dynamics: a virtual that is not only heterogeneous but also embodied. These two aspects of the virtual, the heterogeneous recombination of differential constraints and their embodiment in living systems are, for us, lines of flight from the chain of automatisms of information processing. They are also the two ways, among many other possibilities, through which we have tried to rethink the concept of intensive differential constraints in order to open up towards the production of meaning.

In Sarti, Citti & Piotrowski (2019) we try to address this issue from the point of view of the emergence of the semiotic function. In this work we attempt to show that the saliences of the world and of bodily pregnancies emerge from one and the same heterogenetic process, each instance taking form as a divergent actualization (Deleuze 1988). In other words, starting from the multitude of differential processes that constitute the morphogenetic power of nature, here the actualization of these processes gives rise to the formation of the body and the forms of the world (if we want to see it in a Merleau-Pontyan perspective) and/or gives rise to a stratification of saliences and pregnancies, layers of expression and content (in a Deleuzian perspective). In this last case the body that constitutes itself is a multiple, infra-individual and trans-individual, social body.

In both cases, it must be said, there is a self-production of meaning and, interestingly, no need to invoke a symbolic level in order to speak about meaning. In both cases, divergent actualisation generates spaces, axes on which typical symbolic control devices may only subsequently be installed.

A thousand planes of imaginative ecology

We can define heterogenesis as the set of dynamics of an ecology of immanence where the conditions of possibility are constantly changing – in opposition to a structural dynamics based on systems of oppositions – implemented as systems of attractors or controlled potentials. The emergence of the symbolic is a typical example of structural dynamics that develop around opposition systems, e.g. the semiotic square. Heterogenesis is instead a heterogeneous dynamics that remains pre-symbolic and escapes capture, or has not yet been captured by controlled systems. In this sense it is an ecology of immanence.

Before any other attempt at ecological thinking we must overcome the naive idea of the animal as an entity capable only of automatic stimulus-response. The ecological problem therefore deals, to a certain extent with the problem of automatisms. At a dynamic level you exit from the automatism as soon as you completely access the axis of historicity of

processes. Following Deleuze dynamics takes place on two temporal planes, the plane of Cronos, i.e. the plane of automatic actualization of differential constraints, and the plane of Aion, the plane of access to the past and the possibilities of the future. Liberation from automatism lies in access to this imaginative plane and in the ability to recombine its intensive elements. It is the axis of phylogenesis in the evolution of species, of the invention of the new in cognitive processes, and of the uprising in social dynamics (note that uprising is not revolution, which is instead the structural concept of the transition from one stable state to another).

The planes on which heterogenesis unfolds do not privilege access to humans but are open to an imaginative materialism that is extended to the animal, the vegetable, the inorganic... We are dealing with a generative materiality, capable of creating singularities extending to all scales and constituting a vibrating "chaire" in continuous recombination (Haraway 2016). It is in the multiplicity and diversity of the virtual that we witness a continuous search for the new and a continuous reimagination of the intensive, in contrast to a view of nature as a static system and a repository of immutable laws.

We should thus abandon the reductionist perspective in which the creation of sense would be tied exclusively to the semiolinguistic aspects of human cultural production. Instead, it is necessary to open up towards a much richer primary semiosis, towards an idea of meaningful forms as generated by any encounter between salient forms of the world and bodily, affective pregnancies. This meeting between saliences and pregnancies gives rise to forms of primary signification well before any emergence of the symbolic, as shown by René Thom in a series of works that set-up, in a non-structuralist way his idea of meaning. Both world saliences and bodily pregnancies emerge from the same heterogenetic flow by multiple polarization.

The fact that these semioses are present in every heterogenetic becoming could be important for viewing the relationship between humanity and nature in a different way. It is therefore a question of placing, at the centre of our studies, the conditions of the production of sense that open up the possibility of creating planes of sensitive knowledge extended to technological, social and ecological dimensions.

Always on the side of becoming.

BIBLIOGRAPHY

Bailly, F., & Longo, G. (2011). *Mathematics and the natural sciences: the physical singularity of life.* London: Imperial College Press, (original French version, Hermann, 2006).

Braidotti, R. (2002). *Metamorphoses: Towards a materialist theory of becoming*. Cambridge: Polity Press; Malden, MA: Blackwell.

Capogna, L., Citti, G. & Manfredini, M. (2010). "Smoothness of Lipschitz minimal intrinsic graphs in Heisenberg groups, n>1". *J. Reine Angew. Math*, 648, 75–110.

Citti, G., & Sarti, A. (2006). "A cortical based model of perceptual completion in the roto-translation space". *Journal of Mathematical Imaging and Vision*, 24(3), 307–326.

Deleuze, G. (1994). *Difference and repetition*. New York: Columbia University Press.

Deleuze, G., & Guattari, F. (1987). *A thousand plateaus*. Minneapolis: University of Minnesota Press.

Deleuze, G., & Guattari, F. (1994). *What is philosophy?*. London: Verso.

Favali, M, Citti, G., & Sarti, A. (2017). "Local and global gestalt laws: A neurally based spectral approach". *Neural computation*, 29 (2), 394-422.

Ginzburg, C. (1980). *The Cheese and the Worms: The Cosmos of a Sixteenth Century Miller*. Baltimore: Johns Hopkins University Press.

Gribaudi, M. (2014). *Paris, ville ouvrière. Une histoire occultée (1789-1848)*. Paris, La Découverte.

Haraway, D. (2016). *Staying with the trouble*. Durham: Duke University Press.

Hoffman, W. C. (1989). "The visual cortex is a contact bundle". *Applied Mathematics and Computation*, 32, 137–167.

Hörmander, L. (1967). "Hypoelliptic second order differential equations". *Acta Math*., 119, 147–171.

Hörmander, L. (2009). "Subelliptic Operators". In *The Analysis of Linear Partial Differential Operators IV, Grundlehren der mathematischen Wissenschaften (A Series of Comprehensive Studies in Mathematics)*. Berlin, Heidelberg: Springer.

Longo, G., & Montevil, M. (2014). *Perspective on organisms: Biological time, symmetries, and singularities*. Berlin: Springer.

Nagel, A., Stein, E. M. & Wainger, S. (1985). "Balls and metrics defined by vector fields. I. Basic properties". *Acta Math*., 155. 103–147.

Petitot, J. (2004). *Morphogenesis of meaning*. Bern: Peter Lang.

Petitot J. (1977). "Topologie du carré sémiotique". *Études littéraires*, Volume 10, Numéro 3, 347–426.

Petitot J. (1988). "Approche morphodynamique de la formule canonique du mythe". *L'Homme*, 106-107, 2-3, 24-50.

Petitot, J., & Tondut, Y. (1999). "Vers une neurogéométrie: Fibrations corticales, structures de contact et contours subjectifs modaux". *Mathématiques et Sciences humaines*, 145, 5–101.

Piotrowski, D. (2017). *Morphogenesis of the sign*. Berlin: Springer.

Rothschild, L., & Stein, E.M. (1976). "Hypoelliptic differential operators and nilpotent groups". *Acta Math*., 137, 247–320.

Sarti, A., Citti, G., & Petitot J. (2008). "The symplectic structure of the primary visual cortex". *Biological Cybernetics*, 98, 33–48.

Sarti, A., Citti G., & Piotrowski, D. (2019). "Differential heterogenesis and the emergence of semiotic function". *Semiotica*, Volume 2019, Issue 230, 1-34.

Stengers, I. (2011). *Thinking with Whitehead: a free and wild creation of concepts*. Cambridge: Harvard University Press.

Simondon, G. (2015). *L'individuation à la lumière des notions de forme et d'information*. Paris: J. Millon.

Thom, R. (1972). *Stabilité structurelle et morphogenèse*. New York: Benjamin.

Thom, R. (1980). *Modèles mathématiques de la morphogenèse*. Paris: Christian Bourgois.

Thompson D'A., W. (1917). *On Growth and Form*, Cambridge: CUP, 2nd ed. 1942, repr. 1979.

Turing, A. (1992). *The chemical basis of morphogenesis*. In *Collected Works*, vol. 4, 1–36. Amsterdam: North-Holland. Original paper 1952.